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INTERPHASE MOMENTUM INTERACTION EFFECTS IN THE AVERAGED MULTIFIELD MODEL

PART I: VOID PROPAGATION IN BUBBLY FLOWS

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Abstract—The volume-averaged form of the linear momentum conservation equations for two-phase flow is examined to clarify momentum interaction effects between phases. The case of an accelerating sphere of varying radius in an accelerating fluid is used to derive the form of the interphase force terms. The analysis is extended to assemblies of noninteracting spheres and an interphase force term related to the spatial gradient of phase volume fraction is seen to arise. For the case of bubbly flow, two real characteristics are obtained for dispersed phase volume fractions less than about 0.25. If the term involving the spatial gradient of the phase volume fraction is neglected, then the characteristics are always complex for velocity differences between the phases. The interphase force model is applied to predict experiments on void propagation in bubbly flows. There are no adjustable constants in the model. The experimental data were obtained in our laboratory using cross correlation of signals from a pair of gamma densitometers. The predictions are in excellent agreement with the data. In addition, the predictions are compared with data from several other laboratories, taken over different sets of flow conditions. The predictions are again in close agreement with the data.

1. INTRODUCTION

One of the main approaches to two-phase flow modelling has been to average (in time, space, over an ensemble, or in some combination of these) the original local instantaneous conservation equations, e.g. Agee *et al.* (1978), Banerjee & Chan (1980), Boure (1975), Delhaye & Achard (1978), Drew (1983), Hughes *et al.* (1976), Ishii (1975), Lyczkowski (1978), Nigmatulin (1978), Panton (1968), Vernier & Delhaye (1968), Yadigaroglu & Lahey (1976). In general, a set of averaged conservation equations can be written for each field which may be thought of as a clearly identifiable portion of a phase. For example, annular flow may be modelled by using three fields, one for the droplets, one for the liquid film, and one for the gas core. Selection of the fields depends to some extent on the modeller, but in all cases should be consistent with the physics of the flow situation.

Averaging makes the mathematical aspects of the model much simpler, but information regarding local gradients at interfaces and walls are lost in the process. Consequently closure relationships, sometimes called "constitutive equations", have to be supplied for interfacial and wall transfer of heat, mass, and momentum. In addition, averaging also eliminates information about intraphase distributions of the dependent variables leading to a need for distribution coefficients that relate the product of averages to the average of products.

The form of closure relationships for models with two or more fields may have important consequences on the mathematical structure of the problem. In particular, subtle aspects of momentum interactions between fields play an important role in determining stability. Drew (1983) and several others like Ramshaw & Trapp (1978) have shown that the simplest multifield models proposed, which account for interfacial forces through an algebraic drag correlation, lead to high-wavenumber phenomena that are not physical.

Considerable work has been done to resolve this problem. Stuhmiller (1977) and Banerjee & Chan (1980) have analyzed pressure interactions between the phases and shown that careful consideration of these may lead to more realistic models. Drew *et al.* (1979) have analyzed one important momentum interaction effect, that due to virtual mass, from the viewpoint of material frame indifference. While Drew *et al.* (1979) and Drew & Lahey (1979) have discussed the closure relationships for the linear momentum equation from a continuum-mechanical viewpoint, it is still important to understand the form of the interfield momentum interactions by studying the solution for an actual physical system. To this end, we have selected a problem which may serve to clarify the situation, viz. the forces on an accelerating sphere in an accelerating fluid, and then extended this to an assembly of noninteracting spheres. The main objective of this part of the work is to illustrate the form of the closure relationships that arise in the linear momentum equation.

The results from the model for the interphase forces in bubbly flow are then compared to experiments on void propagation done in our laboratories and by Bernier (1981). Void fraction wave modelling was also investigated by Mercadier (1981) using pipes of hydraulic diameter ~38 mm and liquid superficial velocities ranging from 0 to 1 m/s. These experimental results are in close agreement with ours but the models are different in several respects, e.g. Mercadier accounted for the viscous drag but neglected phasic pressure differences. Our model includes pressure difference effects while neglecting viscous drag. The form used for the virtual mass forces is also different. We shall see that our model is in good agreement with Mercadier's data in addition to the other data.

Micaelli (1982) investigated a somewhat different experimental range using a stochastic approach and taking into account viscous drag and distribution effect. This approach is different from the one presented in our paper.

2 THEORETICAL ASPECTS

2.1 The linear momentum equation

The linear momentum equation for field k has been derived previously, see for example Banerjee & Chan (1980). We briefly recapitulate the form to be used later in the paper.

$$\frac{\partial}{\partial t}\alpha_k\langle\rho_k u_k\rangle+\frac{\partial}{\partial z}\alpha_k\langle\rho_k u_k^2\rangle+\frac{\partial}{\partial z}\alpha_k\langle p_k\rangle-\frac{\partial}{\partial z}\alpha_k\langle\tau_{zz,k}\rangle$$
$$=\alpha_k\langle\rho_k F_k\rangle-\langle m_k u_k\rangle_i-\langle\mathbf{n}_z\cdot\mathbf{n}_k p_k\rangle_i+\langle\mathbf{n}_z\cdot(\mathbf{n}_k\cdot\bar{\bar{\tau}}_k)\rangle_i+\langle\mathbf{n}_{kw}\cdot\bar{\tau}_z\rangle_w\,, \quad [1]$$

where

$$\langle f_k\rangle=\frac{1}{V_k}\int_{V_k}f_k\,\mathrm{d}v,$$
$$\langle f_k\rangle_i=\frac{1}{V}\int_{a_i}f_k\,\mathrm{d}s.$$

α_k is the volume fraction of field k in volume V, $\mathbf{n}_k$ is the outward drawn normal on the surface of field k, $\mathbf{n}_z$ is the unit vector in the z direction, a_i is the interfacial area of field k, and a_{kw} the area of contact between field k and the wall (see also figure 1 in which the

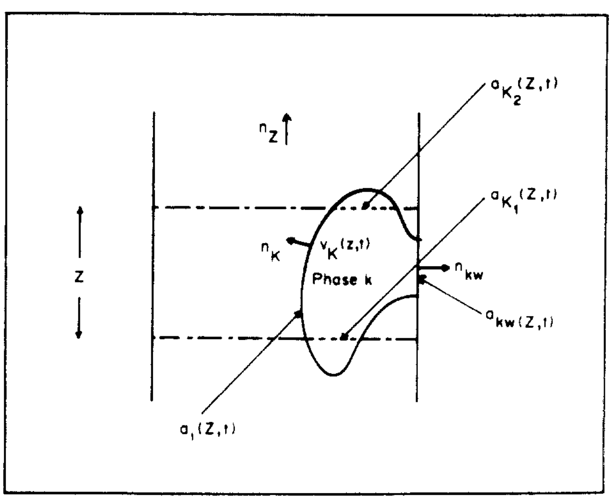


Figure 1 Schematic of two-phase flow defining the symbols

symbols are defined). The other variables are ρ_k, the density of field k; u_k, the velocity in the z direction; p_k, the pressure; $\bar{\bar{\tau}}_k$, the shear stress tensor; F_k, the body force; and $\dot{m}_k$ the mass transfer out of the field.

The pressure term on the right-hand side can be broken into a part that varies over the interface and parts that do not in the following way:

$$p_{ki} = \langle p_k \rangle + \Delta p_{ki} + \Delta p'_{ki}\,, \qquad [2]$$

with

$$\Delta p_{ki} = \frac{V}{a_i}\langle p_{ki}\rangle_i - \langle p_k \rangle\,,$$

which does not vary over a_i in V, and

$$\Delta p'_{ki} = p_{ki} - \frac{V}{a_i}\langle p_{ki}\rangle_i\,.$$

To proceed, we recapitulate a form of Gauss' theorem particular to figure 1 (see also Delhaye & Achard (1978) for derivation). Gauss' theorem applied to volume $V_k(z,t)$ is

$$\int_{V_k(z,t)} \nabla\cdot\mathbf{A}\,\mathrm{d}V = \int_{a_i(z,t)} n_k\cdot\mathbf{A}\,\mathrm{d}S - \int_{a_{k1}(z,t)} \mathbf{n}_z\cdot\mathbf{A}\,\mathrm{d}S + \int_{a_{k2}(z,t)} \mathbf{n}_z\cdot\mathbf{A}\,\mathrm{d}S + \int_{a_{kw}(z,t)} \mathbf{n}_{kw}\cdot\mathbf{A}\,\mathrm{d}S\,,$$

where the symbols are defined in figure 1, and $\mathbf{A}$ is any vector field.

From the definition of a volume integral

$$\frac{\partial}{\partial z}\int_{V_k(z,t)} A_z\,\mathrm{d}V = \frac{\partial}{\partial z}\int_{z-Z/2}^{z+Z/2}\left(\int_{a_k(z')} A_z\,\mathrm{d}S\right)\mathrm{d}z'\,,$$

where $A_z = \mathbf{n}_z\cdot\mathbf{A}$. Noting that

$$\frac{\mathrm{d}}{\mathrm{d}\sigma}\int_{a(\sigma)}^{b(\sigma)} f(r)\mathrm{d}r = b'(\sigma)f[b(\sigma)] - a'(\sigma)f[a(\sigma)]\,,$$

we have

$$\frac{\partial}{\partial z}\int_{V_k} \mathbf{n}_z\cdot\mathbf{B}\,\mathrm{d}V = \int_{a_{k2}(z,t)} \mathbf{n}_z\cdot\mathbf{B}\,\mathrm{d}S - \int_{a_{k1}(z,t)} \mathbf{n}_z\cdot\mathbf{B}\,\mathrm{d}S\,,$$

from which it follows that

$$\int_{V_k(z,t)} \nabla\cdot\mathbf{A}\,\mathrm{d}V = \frac{\partial}{\partial z}\int_{V_k(z,t)} \mathbf{n}_z\cdot\mathbf{A}\,\mathrm{d}V + \int_{a_i(z,t)} \mathbf{n}_k\cdot\mathbf{A}\,\mathrm{d}S + \int_{a_{kw}(z,t)} \mathbf{n}_{kw}\cdot\mathbf{A}\,\mathrm{d}S\,.$$

If $\mathbf{n}_{kw}$ is perpendicular to the direction of motion, it follows immediately that

$$\frac{1}{V}\int_{a_i} \mathbf{n}_k\cdot(\mathbf{n}_z p_k)\mathrm{d}s = -\left[\langle p_k\rangle + \Delta p_{ki}\right]\frac{\partial \alpha_k}{\partial z} + \langle \mathbf{n}_k\cdot\mathbf{n}_z\cdot\Delta p'_{ki}\rangle_i$$

from the definition of $\langle p_k \rangle$ and Δp_{ki}.

The same analysis could be performed on the term involving $\bar{\bar{\tau}}_k$ on the rignt-hand side of [1], but it is sufficient for our purpose to consider the pressure term. The linear momentum equation then becomes

$$\frac{\partial}{\partial t}\alpha_k\langle\rho_k u_k\rangle + \frac{\partial}{\partial z}\alpha_k\langle\rho_k u_k^2\rangle + \alpha_k\frac{\partial\langle p_k\rangle}{\partial z} - \frac{\partial}{\partial z}\alpha_k\langle\tau_{zzk}\rangle$$

$$= \Delta p_{ki}\frac{\partial\alpha_k}{\partial z} + \langle\alpha_k\rho_k F_k\rangle - \langle\dot{m}_k u_k\rangle_i + \langle\mathbf{n}_k\cdot\bar{\tau}_z\rangle_i - \langle\mathbf{n}_k\cdot\mathbf{n}_z\,\Delta p'_{ki}\rangle_i + \langle\mathbf{n}_{kw}\cdot\bar{\tau}_z\rangle_w\,. \quad [3]$$

Several interesting aspects are apparent in this equation. For example, $\langle p_k\rangle$ is the average pressure in field k and may vary from field to field, e.g. due to gravitational and surface tension effects in stratified flows. Δp_{ki} is the difference between the average pressure in field k and the average pressure at the interface within field k, and may be significant in many situations, e.g. again in stratified flows due to gravitational effects or in dispersed flows where the average pressure at the interface is different from the average pressure in the phase. Another important term is $\langle\mathbf{n}_k\cdot\mathbf{n}_z\,\Delta p'_{ki}\rangle_i$ which gives the force per unit volume in the z direction due to pressure variations over a_i. For accelerating flows, this is significant even if the phases are considered to be inviscid. For nonaccelerating viscous flow, this term leads to the form drag. On the other hand, for stratified flow with no waves, this term vanishes since there are no pressure variations on the interface.

We will now proceed to calculate the form of the terms on the right-hand side of [3] for the case of an accelerating sphere of varying radius in an accelerating incompressible inviscid fluid. We will assume no interfield mass transfer and no wall effects to simplify matters.

2.2 Forces on an accelerating sphere in an accelerating fluid

Consider the forces acting on an accelerating sphere of varying radius in an accelerating inviscid fluid. The sphere may be thought of as an expanding or collapsing gas bubble; all variables associated with the sphere will be subscripted with G, the variables associated with the surrounding fluid will be subscripted with L.

The following discussion will clarify the methodology. We will then extend the analysis to an assembly of spheres and examine the effect of spatial gradients in phase volume fractions.

To simplify the notation we will drop the averaging signs in the following with the understanding that the averaging volume is large compared to the volume of the sphere.

Starting with a simple case to illustrate the main effects, consider a fixed sphere (i.e. $u_G = 0$) in a large body of accelerating fluid. The potential describing the flow is given by Milne-Thompson (1968) as

$$\phi = u_L\left(r + \frac{R^3}{2r^2}\right)\cos\theta\,. \quad [4]$$

R, u_L, and (r, θ) are, respectively, the radius of the sphere, the fluid velocity far from the sphere, and polar coordinates referred to the center of the sphere.

The unsteady Bernoulli equation gives the pressure in the flow field and in particular at the sphere–fluid interface as

$$p_{Li} = p_{Lo} + \frac{3}{2}\rho_L R\,\dot{u}_L(\cos\theta - 1) - \frac{9}{8}\rho_L u_L^2\sin^2\theta\,. \quad [5]$$

p_{Lo} is the pressure at the front stagnation point (i.e. at $\theta = 0$). Using an averaging volume that encompasses the sphere, it is straightforward to show that the integrated interfacial force is

$$\langle\mathbf{n}_L\cdot\mathbf{n}_z\,\Delta p'_{Li}\rangle_i = \frac{3}{2}\alpha_G\rho_L\dot{u}_L\,. \quad [6]$$

In this problem the fluid accelerates uniformly, i.e. there is no spatial velocity gradient. Hence in [6], $\dot{u}_L$ is the derivative of u_L with respect to time.

Consider now the more general case of an accelerating sphere of variable radius in an accelerating inviscid fluid. To simplify matters, let the flow be without circulation. In this case, the force over the sphere can be found by using the Cauchy–Lagrange equation which is similar to Bernoulli's equation except for an additional term which arises if the potential is referred to a moving frame of reference (see also Yakimov 1971 and Voinov 1973). The basis of the following derivation is that the sphere introduces a small perturbation into the undisturbed potential for the continuous phase flow field.

The Cauchy–Lagrange integral in a system moving with velocity u_G, in which the motion of the fluid is described by the potential ϕ, has the form

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}(\nabla \phi)^2 - u_G \cdot (\nabla \phi) + \frac{p}{\rho} + U = f(t) . \qquad [7]$$

Omitting the potential of external forces U, the force exerted by the flow on the sphere, in the z direction, is

$$F_z = \rho \int_s \left(\frac{\partial \phi}{dt} + \frac{1}{2}(\nabla \phi)^2 - u_G \cdot (\nabla \phi) \right) \mathbf{n}_k \cdot \mathbf{n}_z \, \mathrm{d}S . \qquad [8]$$

Consider now some arbitrary potential flow ϕ_0 devoid of singularities. ϕ_0 can be written in terms of its infinite Taylor's expansion around $y_i = q_i$:

$$\phi_0 = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{\partial^n \phi_0}{\partial y_i \cdots \partial y_k} x_i \cdots x_k,\ x_i = y_i - q_i,\ i, j, \cdots, k = 1, 2, 3 .$$

The y_i are the absolute Cartesian coordinates, the x_i are Cartesian coordinates relative to the system centered on $(q_i(t))$. ϕ_0 satisfies continuity, therefore each term of this series is a harmonic function.

We perturb the flow field by introducing an infinitesimal sphere at $q_i(t)$ moving with velocity $u_G = \dot{q}_i$.

Neglecting the far boundaries, ϕ must have singularities only within the sphere and tend to ϕ_0 at infinity. The boundary condition at R is

$$\left. \frac{\partial \phi}{\partial r} \right|_R = \frac{u_{Gi} x_i}{R} + \dot{R} .$$

The unique harmonic function satisfying these conditions is

$$\phi = \phi_0 - \frac{R^2 \dot{R}}{r} - \frac{u_{Gi} x_i R^3}{2r^3} + \sum_{n=0}^{\infty} \frac{1}{n!} \frac{n}{n+1} \frac{R^{2n+1}}{r^{2n+1}} \frac{\partial^n \phi_0}{\partial y_i \cdots \partial y_k} x_i \cdots x_k , \qquad [9]$$

where

$$r^2 = x_i x_i .$$

This equation holds only if the distance to the flow boundary is large compared to the size of the sphere. For example, introducing a sphere in a still body of liquid with a planar wall, the potential would be formed with the image locations to the wall discontinuity.

The first term of the integral [8] can be evaluated exactly by noticing that terms of order $n > 1$ are orthogonal to x_z:

$$\int_s \frac{\partial \phi}{\partial t} \mathbf{n}_k \mathbf{n}_z \, \mathrm{d}S = \frac{4\pi R^3}{3} \frac{D_G u_{Lz}}{Dt} + \frac{2\pi}{3} \frac{D_G}{Dt} (R^3 (u_{Lz} - u_{Gz})) . \qquad [10a]$$

The material derivative

$$\frac{D_k}{Dt} = \frac{\partial}{\partial t} + u_k \frac{\partial}{\partial z}.$$

Dropping terms of order R^5 and above gives the two last terms in [8] as

$$\int_s \left[\frac{1}{2}(\nabla\phi)^2 - u_G \cdot (\nabla\phi)\right] \mathbf{n}_k \cdot \mathbf{n}_z \, dS = -2\pi R^3 \left((u_G - u_L) \cdot \nabla\right) u_{Lz}. \qquad [10b]$$

Combining [10a] and [10b] and adding the buoyancy force arising from the potential U gives the force on a sphere small compared to the flow field

$$F = \rho_L \alpha_G \left[\frac{1}{2}\left(\frac{D_L u_L}{Dt} - \frac{D_G u_G}{Dt}\right) + \frac{3}{2R}(u_L - u_G)\frac{D_G R}{Dt} + \frac{D_L u_L}{Dt} - g_z\right]. \qquad [11]$$

According to this derivation, the equation of motion for the sphere is

$$\alpha_G \rho_G \frac{D_G u_G}{Dt} = +\alpha_G \rho_L \left[\frac{1}{2}\left(\frac{D_L u_L}{Dt} - \frac{D_G u_G}{Dt}\right) + \frac{3}{2R}(u_L - u_G)\frac{D_G R}{Dt} + \frac{D_L u_L}{Dt} - g_z\right]. \qquad [12]$$

Note that if $u_G = 0$ and R is constant, then the form of [12] reduces exactly to [6], with $\dot{u}$ clearly being given by $D_L u_L / Dt$. Note that to the level of our approximation u_L is the undisturbed velocity of the continuous phase flow field if the sphere were absent.

The physical significance of the terms in [12] are as follows:

— the first term on the right-hand side is the "virtual mass" force due to relative acceleration between the phases;

— the second term is a force due to radius change;

— the third term is the "virtual buoyancy" which arises out of the pressure gradient in the continuous phase, as will be demonstrated later;

— the fourth term is the buoyancy force.

To further assess the validity of [12], consider the case when the spatial liquid velocity gradient is important, e.g. in the case of a sphere placed in the flow field of a source. Then the force exerted by the flow field on a fixed sphere can be shown to be an attraction towards the source of magnitude (see for example Milne-Thompson 1968):

$$F = \frac{4\pi\rho m^2 a^3}{r(r^2 - a^2)^2},$$

where r is the distance from the sphere center to the source, a the radius of the sphere, and m the source strength. If a is small compared to r, which is consistent with our assumption for [12], then the far-field liquid velocity potential can be approximated to that of a source of strength m, and it is found that the force is

$$\frac{3}{2}\alpha_G \rho_L u_L \frac{du_L}{dr}. \qquad [13]$$

This is exactly what would be obtained from [12] if $u_G = 0$, R is constant, and $u_L = u_L(r)$. Therefore, we conclude that the results of the preceding analysis reduce to the correct limits.

We will not consider in more detail the last two terms in [12]. To clarify how these terms arise, we note that the equation of motion for the continuous phase is

$$\rho_L \frac{D_L u_L}{Dt} - \rho_L g_z = -\frac{\partial p_L}{\partial z}. \tag{14}$$

This equation shows that the buoyancy and virtual buoyancy terms in [12] are in fact the pressure gradient of the continuous phase impressed on the sphere. This is consistent with the way the momentum equation was derived. Equation [12] indicates that the pressure gradient is contained in the interfacial terms when the averaging volume encompasses the sphere. Combining [14] and [12] gives the momentum equation for dispersed flows of low void fraction:

$$\alpha_G \rho_G \frac{D_G u_G}{Dt} + \alpha_G \frac{\partial p_L}{\partial z} = \alpha_G \rho_L \left[\frac{1}{2}\left(\frac{D_L u_L}{Dt} - \frac{D_G u_G}{Dt}\right) - \frac{3}{2R}(u_G - u_L)\right]\frac{D_G R}{Dt} + \alpha_G \rho_G g_z. \tag{15}$$

This equation agrees with Nigmatulin (1979) for dispersed systems with low void fraction (when bubble interaction effects may be neglected) though it is derived in a completely different way.

2.3. *Forces on an assembly of spheres*

Consider now an assembly of spheres of constant radius for a velocity with mean fluid velocity in the z direction, as shown in figure 2. An infinitesimally thick control volume cuts the spheres, as shown in the figure. We will assume that the spheres are sufficiently far apart that the interactions between them are weak. Also, we will assume that they are placed relatively randomly but that there are spatial (and temporal), gradients in phase volume fraction. To simplify matters further, we take the average diameter of the spheres to be constant.

The derivation of the averaged-momentum equations is straightforward if it is recognized that the sum of the interfacial pressure forces acting on the segments of the spheres

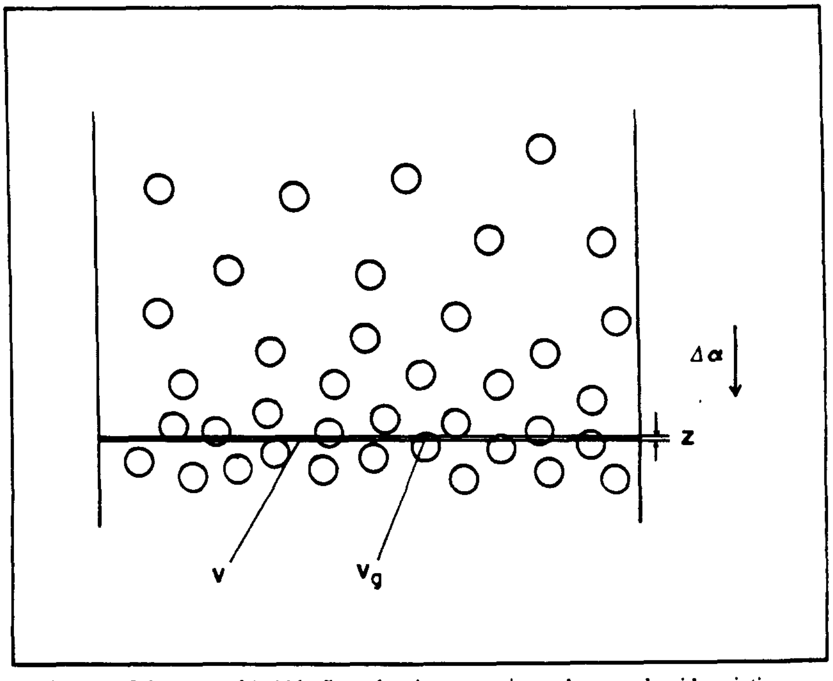


Figure 2. Schematic of bubbly flow showing averaging volume and void variation.

intersecting the control volume is equivalent to the integral of the pressure variations over the surface of a single sphere per unit interfacial area times the interfacial area per unit volume. The virtual buoyancy is now contained in the pressure gradient terms and the momentum conservation equations are, from [3],

$$\rho_G \alpha_G \frac{D_G u_G}{Dt} + \alpha_G \frac{\partial p_G}{\partial z} = \Delta p_{Gi} \cdot \frac{\partial \alpha_G}{\partial z} - \frac{1}{2} \rho_L \alpha_G \left(\frac{D_G u_G}{Dt} - \frac{D_L u_L}{Dt} \right) + \alpha_G \rho_G g_z, \quad [16]$$

$$\rho_L \alpha_L \frac{D_L u_L}{Dt} + \alpha_L \frac{\partial p_L}{\partial z} = \Delta p_{Li} \cdot \frac{\partial \alpha_L}{\partial z} + \frac{1}{2} \rho_L \alpha_G \left(\frac{D_G u_G}{Dt} - \frac{D_L u_L}{Dt} \right) + \alpha_L \rho_L g_z \quad [17]$$

Notice the difference between [15] and [16], putting aside the fact that [15] accounts for a variable bubble radius. In [15] the averaging volume encompasses the sphere; therefore, in general (that is, when the sphere is not crossing the averaging volume boundary), the effect of the liquid net pressure gradient must be contained in the interfacial term, and the void fraction gradient vanishes. In [16], the effect of the net liquid pressure gradient is contained in the gas pressure gradient, and the void fraction gradient appears naturally from [3].

In writing this form of the momentum equations we have assumed that distribution coefficients are 1.0, i.e. products of averages are equal to the average of products. The coefficient for the last term on the right-hand side of [16] and [17] is based on there being no interactions between the spheres. In general, this coefficient will depend somewhat on phase volume fraction.

To proceed, we need to calculate Δp_{Li} and Δp_{Gi}, recalling that these are differences between the average pressure at the interface and the average pressure within each phase. It is a good assumption to put $\Delta p_{Gi} = 0$ since the average pressure within the spheres will be very close to the average interfacial pressure. To obtain Δp_{Li} it is sufficient to find the average interfacial pressure, say from [5] or its equivalent. For spheres of constant radius, the integration yields

$$\Delta p_{Li} = -\frac{1}{4} \rho_L (u_G - u_L)^2 . \quad [18]$$

Together with the mass conservation equations for the dispersed and continuous phase, the momentum conservation equations now form the model we will use for analysis of the experimental results. Note that we will use expressions for the virtual mass coefficient and Δp_{Li} that assume no interactions between spheres. The virtual mass terms in the model are exactly the same as derived by Nigmatulin (1979) for assemblies of spheres. The term involving the spatial gradient of the phase volume fraction has not been used previously, to our knowledge.

2.4 Characteristics and void propagation

The momentum and mass conservation equations form a quasilinear model with first-order derivatives. The viscous terms are usually modelled through "drag" or "friction" correlations which are algebraic functions of the dependent variables. These do not affect the velocities of propagation of disturbances. There are also transient viscous effects, e.g. the Basset force, which involve history integrals. To a first approximation, these terms do not affect phase velocities in the linear dispersion relationship. Therefore, it is sufficient for our purposes to consider only the derivative terms in the model. Also, for this case $\rho_G << \rho_L$. The characteristics λ are then determined from

$$\det \begin{vmatrix} -\alpha_G & 0 & \lambda - u_G & 0 \\ 0 & -\alpha_L & -(\lambda - u_L) & 0 \\ \{\rho_L C_{VM} (\lambda - u_G) + 1/2 \rho_L (u_G - u_L)\} & \{-\rho_L C_{VM} (\lambda - u_L) - 1/2 \rho_L (u_G - u_L)\} & 0 & -1 \\ \rho_L \alpha_G C_{VM} (u_G - \lambda) & (\rho_L \alpha_L + \rho_L C_{VM} \alpha_G)(\lambda - u_L) & -\Delta p_{Li} & -\alpha_L \end{vmatrix} = 0 . \quad [19]$$

Here we have left C_{VM}, the virtual mass coefficient, and Δp_{Li} unspecified. $C_{VM} = \frac{1}{2}$ and $\Delta p_{Li} = -\frac{1}{4}\rho_L(u_G - u_L)^2$ for the case of noninteracting spheres. And the characteristics for void propagation are given by

$$\lambda^* = \frac{\alpha_L}{2 + 4\alpha_G\alpha_L} \pm \frac{\sqrt{\Delta}}{1 + 2\alpha_L\alpha_G},$$

where

$$\lambda^* = \frac{\lambda - u_L}{u_G - u_L}, \quad [20]$$

$$\Delta = \frac{1}{4}\alpha_L^2 - \alpha_G\alpha_L\left(\frac{1}{2} + \alpha_G\alpha_L\right). \quad [21]$$

Note that for $\alpha_L \sim 0.74$, $\Delta \sim 0$. Therefore, the characteristics are wholly real only for

$$\alpha_L > 0.74 \text{ (or } \alpha_G < 0.26).$$

Thus it would be expected that the coefficients in the interphase force terms would have to be changed to maintain real characteristics outside this range of void fraction. This could arise due to flow regime transition. Interestingly enough, this flow regime transition (if, indeed, a change to complex characteristics signals such a transition) is independent of physical properties and only depends on the phase volume fraction for ($\rho_G << \rho_L$).

λ^* is plotted against α_G in figure 3. As mentioned previously, these characteristics are for void propagation—the phases are assumed incompressible so pressure waves propagate infinitely fast. Pressure wave behavior will be the subject of a subsequent paper because strongly nonlinear effects enter through the relationship for phase pressure difference, and thermal effects have to be considered very carefully. For large amplitude pressure waves, the analysis has to be altogether more sophisticated.

The results in figure 3 are correct to the following level of approximation in distribution effects:

$$\overline{\alpha_k\langle u_k^2\rangle} = \overline{\alpha_k}\,\overline{\langle u_k\rangle^2}, \quad [22]$$

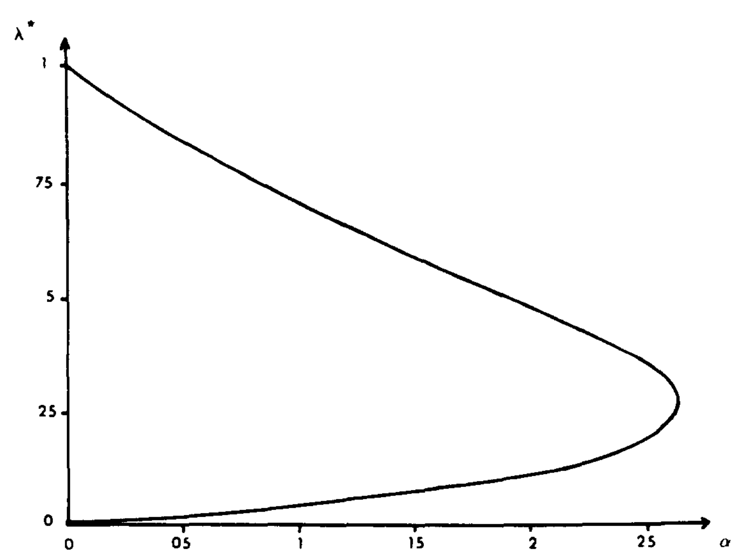


Figure 3. The characteristics λ^* as a function of void fraction for bubbly flows.

where

$$\overline{\langle u_k \rangle} = \frac{\overline{\alpha_k \langle u_k \rangle}}{\overline{\alpha_k}}$$

and the overbar denotes a time or ensemble average (see also Banerjee & Chan 1980)

3 COMPARISON WITH EXPERIMENTS

3.1 Experimental equipment

Experiments were done in the vertical air–water loop shown schematically in figure 4. The piping in the test section was 38 mm I.D. and the first measurement station was 5 m from the air inlet, which was a mixing tee.

In addition to flow rate measurement for each phase, the cross-section averaged void fraction was measured at two stations placed 62 mm apart. The void fraction was measured with a gamma densitometer, the design of which is discussed by Chan & Banerjee (1981). 300 mCi Americium sources were used; typical count rates were 30,000 counts/s with pipe full of water, and 50,000 counts/s with the pipe empty. Gamma beams of the same width as the pipe were used to reduce flow regime dependence on the count rate for a particular void fraction. Nonetheless, the densitometers were calibrated with Lucite pieces of various shapes. The calibration curves are shown in figure 5. The densitometer signals were fed to a cross correlator. The real and imaginary part of the cross spectrum, and hence the phase lag, was found. The technique is extensively discussed in the papers by Heidrick *et al.* (1977) and will not be considered here. The measuring stations were close enough that the coherence between the signals was ~1.0 in the range of frequencies of interest. This indicated that structures were propagating between the densitometers in a relatively undistorted form. Consequently, phase velocities could be calculated from the phase lags as a function of

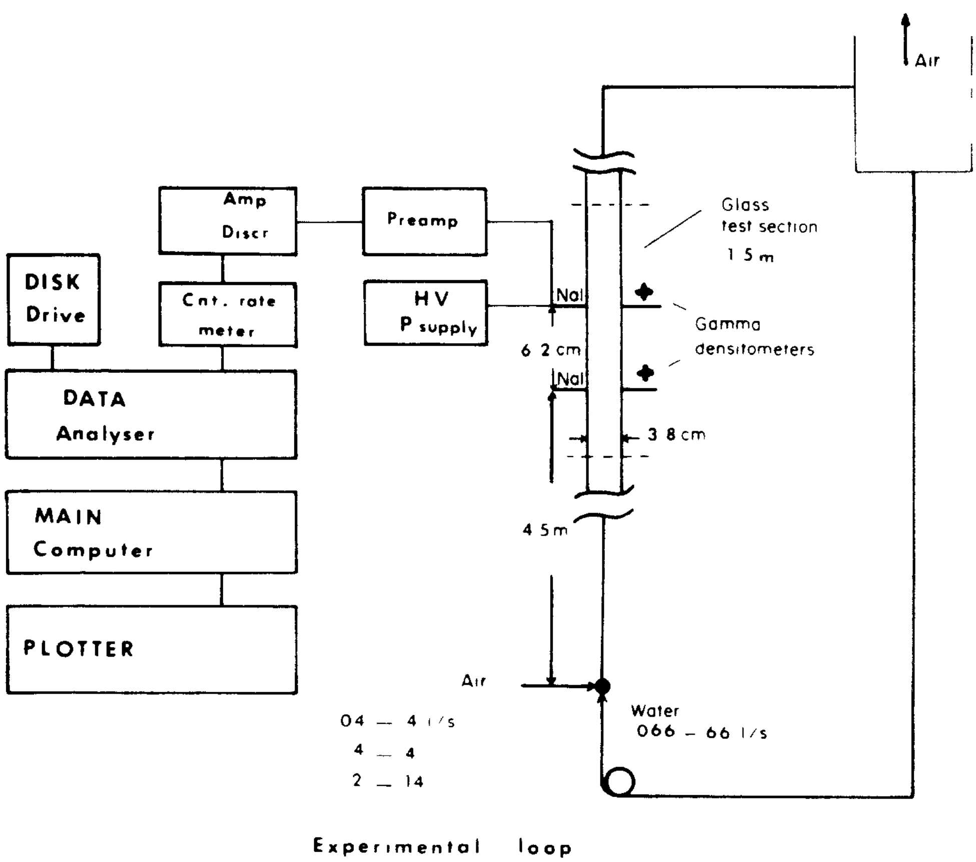


Figure 4. Schematic of experimental apparatus used for void propagation studies.

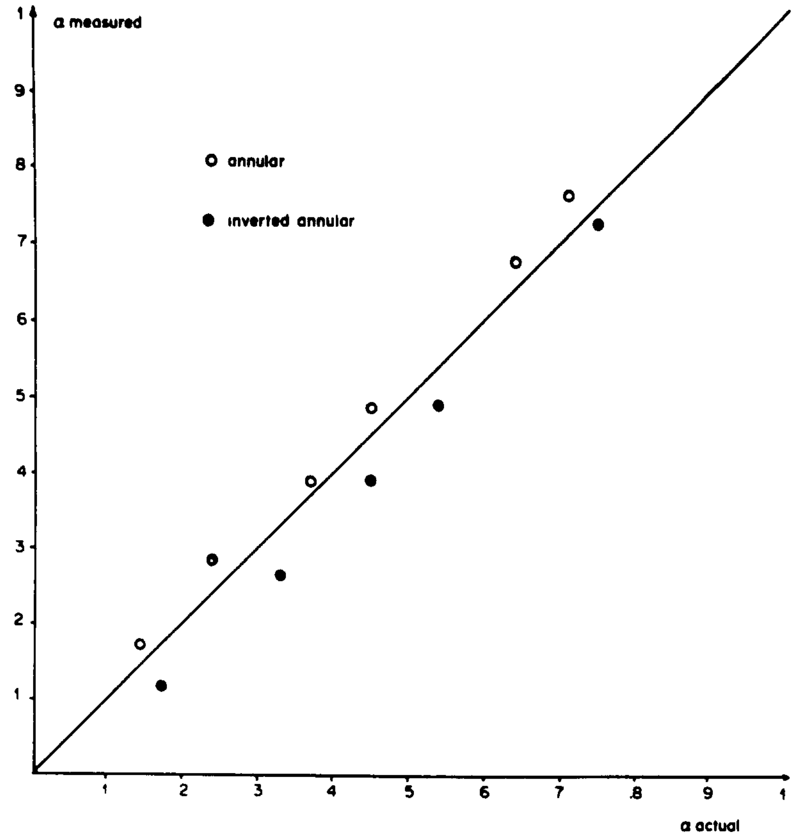
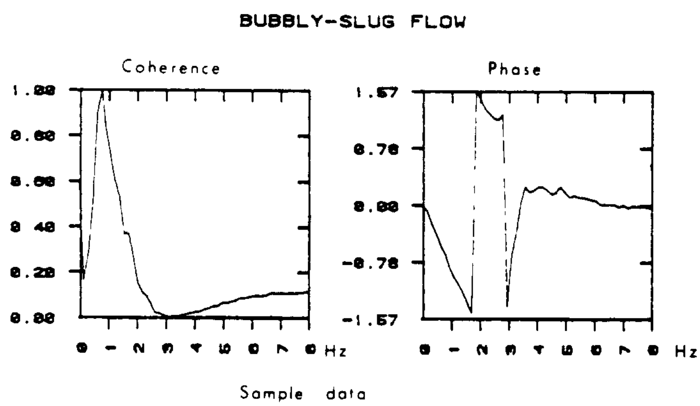


Figure 5 Calibration points for the gamma densitometer for different flow regimes simulated with Lucite pieces.

frequency. A typical phase lag vs. frequency plot for the bubbly-slug flow region is shown in figure 6 along with the coherence function. Note that the data is centered about the higher value of the propagation velocity (if there are two distinct values as suggested by the analysis) as this will have a higher coherence. Coherence drops off with propagation time between the measuring stations, so the data will tend to be biased towards identification of the higher void propagation velocity.



Sample data

Liquid velocity	.104 m/s
Gas " "	66
Void " "	42
Frequency	781 Hz
Coherence	97

Figure 6. Typical coherence and phase spectra. The phase spectrum is linear at low frequency indicating that the system is nondispersive.

Data were taken in the bubbly-slug, slug, churn, and annular flow regions. We will only discuss the bubbly-slug flow data in this paper.

Bernier's (1981) experiments. Bernier (1981) did an extensive series of experiments at Caltech under C. Brennen's supervision. The test section was 101.6 mm I.D. and the first measuring station was 1 m from the inlet. Cross-section averaged void measurements were made by two impedance void meters. They operate on the principle that the bulk electrical impedance of the mixture is usually different from the impedance of each constituent The technique has been used and discussed by Orbeck (1962) and Olsen (1967)

Bernier's data cover a range of bubbly flows occurring at low water velocity with bubble diameters on the order of 0.5 cm and void fractions ranging from 0 to 25%

3.2 Predictions of data

The model proposed in the theoretical section was applied to our own data and the data of Bernier (1981). Our data were taken with $0.1 \le \alpha_G \le 0.15$. In these experiments, the superficial gas velocity j_G was kept constant ($\sim$ 0.1 m/s) and the liquid velocity was varied. The superficial liquid velocity was $0.0884 \le j_L \le 0.765$ m/s. The results for the bubbly flow experiments are shown in figure 7. The agreement between the experiments and the model is remarkable.

Bernier's data cover a narrower range of liquid superficial velocities, $0 \le j_L \le 0.318$ m/s, but a wider range of phase volume fractions. Figure 8 compares Bernier's results with our predictions. Again, the predictions are in remarkable agreement with the data.

Mercadier (1981) measured wave velocity using the cross-correlation function. The experiments cover a comparable range of liquid and gas superficial velocities. Void fraction wave velocities lie in between the gas and liquid velocities as predicted by the model. Mercadier's data is compared with the predictions of our model in figure 9.

Micaelli (1982) considers bubbly flow regimes with high liquid velocity and smaller bubble radius (0.5–2.5 mm). In this region, the relative velocity is small and the liquid velocity is sometimes greater than the gas velocity due to distribution effects. As shown in figure 10, Micaelli's data is well predicted. However, the good fit between Micaelli's data and our model may not assess its validity since the relative velocity between the gas and liquid is much smaller than for Bernier's or our data.

4 CONCLUSIONS

The form of the forces on spheres in an inviscid flow with no circulation has been derived, assuming no interaction between the spheres. The expression is very similar to that of Nigmatulin (1979) except that an additional term related to the spatial gradient of the

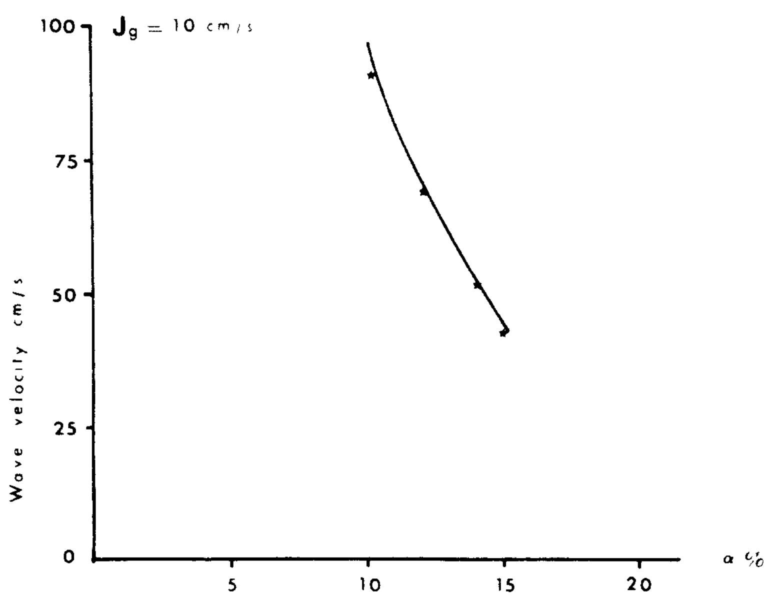


Figure 7 Void wave velocity as a function of α_G for a given superficial velocity in our experiments Each point represents a different liquid superficial velocity.

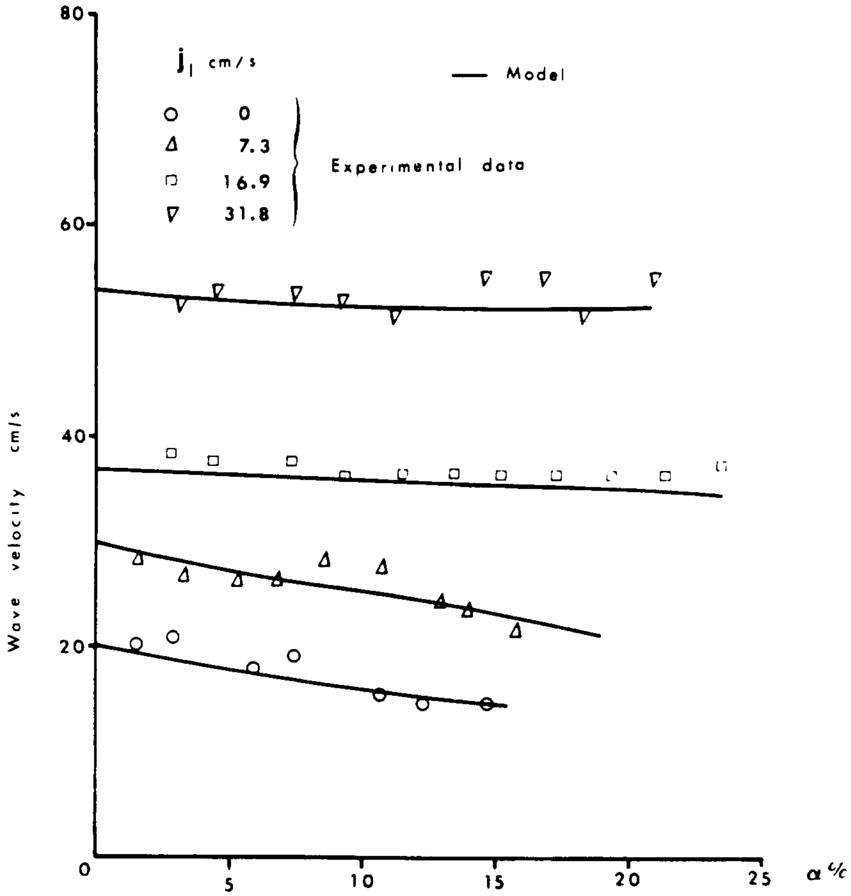


Figure 8 Void wave velocity as a function of α_G for different values of the liquid superficial velocity from Bernier (1981).

phase volume fraction arises. This term is crucial in the analysis of void propagation in bubbly flows.

Based on the reasonable assumption that viscous effects enter the momentum equations primarily through algebraic terms, the characteristics for void propagation have been determined. For theoretical values of the coefficients for the forces in inviscid flow, assuming no interaction between spheres, it is found that the characteristics are wholly real for

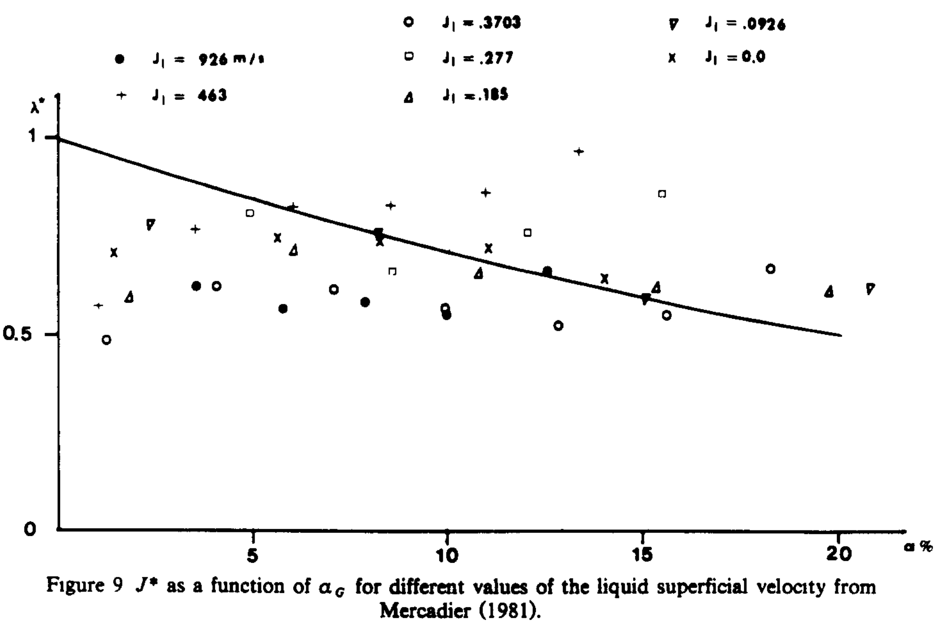


Figure 9 J^* as a function of α_G for different values of the liquid superficial velocity from Mercadier (1981).

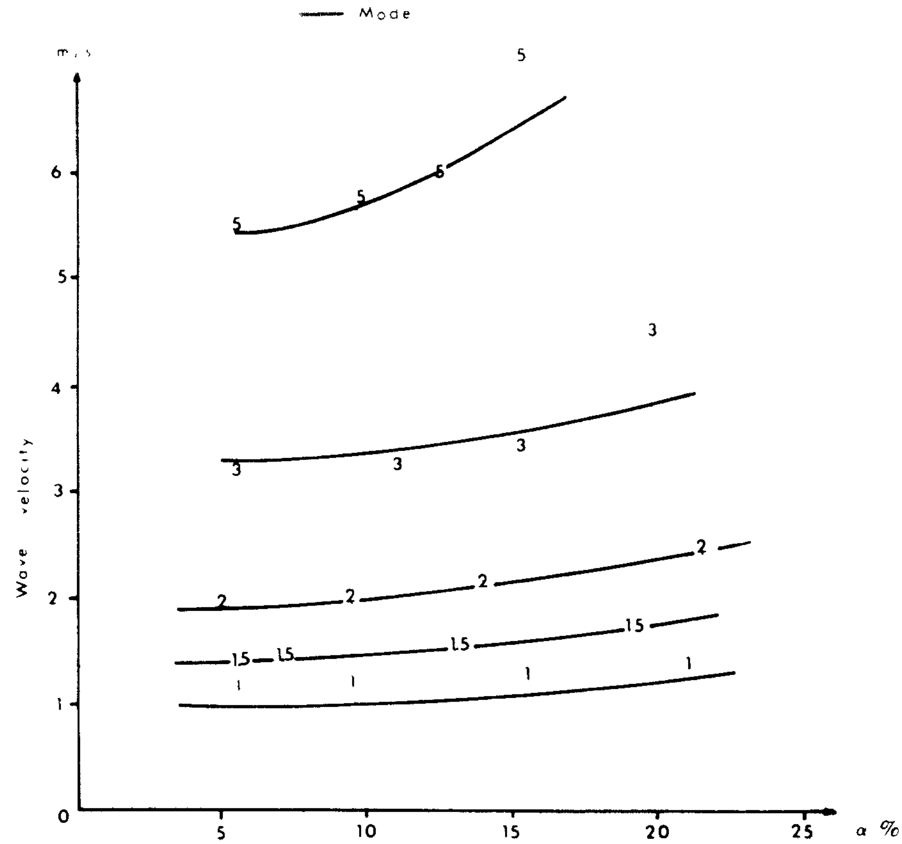


Figure 10. Comparison of model with Micaelli's data. The numbers indicate the value of the liquid superficial velocity Void wave velocity is plotted as a function of α_G.

$\alpha_G < 0.26$. This result is independent of flow rates and physical properties but requires $\rho_G << \rho_L$.

The predictions have been compared with our own data and that obtained in other laboratories. The agreement is excellent and enhances confidence in the proposed model.

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